Complex Analysis Study Guide Test 3

The test is Monday April 24. No calculators. Closed notes.

Material for test 3:

- Everything from the 2nd test to this week.
- Results from Cauchy's Integral Formula
- Complex Sequences
- Taylor Series
- Laurent Series
- Singularities
- Analytic Continuation
- Residue Theory
- Real integrals on $(-\infty, \infty)$.

Practice problems

Below are a selection of problems from our textbooks that looks like reasonable problems that could appear on a test. An "easy" problem means that you should be able to jump right in and start solving it immediately. A "medium" problem means it is expected that you'll need to think a little before solving the problem. A "hard" problem means you'll need to think a lot and maybe work out some details before solving the problem.

Easy Problems

1. Find and describe the singularities and residues of $f(z) = \frac{e^{2z}-1}{z^2}$ (R3.5.1.b)
2. Find and describe the singularities and residues of $f(z) = \log\left(1 + z^{\frac{1}{2}}\right)$ (R3.5.1.f)
3. Find and describe the singularities and residues of $f(z) = \frac{z}{\sin^2(z)}$ (R3.5.3.c)
4. Where on the plane can the function $f(z) = \sum_{k=0}^{\infty} z^k$, $|z| < \frac{1}{2}$, be analytically continued to?
5. Find a closed form expression for $\sum_{k=1}^{\infty} z^k(1-z)$ (6.2)
6. Find the region of convergence of $\sum_{k=1}^{\infty} \frac{z^k}{k(k+1)}$ (6.10)
7. Find the region of convergence of $\sum_{k=1}^{\infty} \frac{(z+2)^{k-1}}{(k+1)^3 4^k}$ (6.12)
8. Find the series expansion for $\frac{e^{2z}}{(z-1)^3}$ centered at $z = 1$. (6.26a)
9. Find the series expansions for $\frac{1}{(z+1)(z+3)}$ centered at 1. (Three different answers that cover almost the entire plane) (6.27a)
10. Find the residue of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at each of its poles. (7.4)
11. Find $\int_C \frac{e^{zt}}{z^2(z^2+2z+2)} dz$ where $C = \{z \in \mathbb{C}: |Z| = 3\}$ (7.6)
12. Find $\int_{-\infty}^{\infty} \frac{1}{x^2+49} dx$ using complex analysis. (R4.2.1.a)
13. Find $\int_0^{\infty} \frac{1}{x^6+1} dx$ using complex analysis (7.9)
14. Find $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2x+2)} dx$ using complex analysis. (7.10)
15. Find $\int_{-\infty}^{\infty} \frac{x\sin(\pi x)}{x^2+2x+5} dx$ using complex analysis. (7.17)

Medium problems

1. Find $\int_0^{\infty} \frac{\ln(x^2+1)}{x^2+1} dx$ using complex analysis. (7.22)
2. Where on the plane can the function $f(z) = \sum_{k=0}^{\infty} z^{4^k}$, $|z| < \frac{1}{2}$ be analytically continued to? (R3.5.3.c)
3. Find the series expansion for $\log(1+z)$ without using Taylor series. (6.23)
4. Find the series expansion for $\frac{z}{(z+1)(z+2)}$ centered at $z = -2$ (6.26d)
5. Find the series expansion for $\frac{e^{2z}}{(z-1)^3}$ centered at $z = 2$.
6. Find the series expansion for $\frac{e^{2z}}{(z-1)^3}$ centered at $z = 0$.
7. Find the closed form expression for $-\sum_{k=0}^{\infty}(k+1)^n$ (R3.2.5.a)
8.
9.

Hard problems

1. Find $\int_C \frac{\left(z+\frac{1}{z}\right)^{2n}}{z} dz$ where C is the unit circle. (R3.2.10)